FORMULARI DE FÍSICA QUÀNTICA

Víctor García Carrasco

CONSTANTS

$h = 6,62608\times10^{-34}\, J\cdot s \quad \hbar = 1,05459\times10^{-34}\, J\cdot s$

$e = 1,60219\times10^{-19}\, C$

$k = 1,38066\times10^{-23}\, J\cdot K^{-1}$

$\sigma = 5,67051\times10^{-8}\, W\cdot m^{-2}\cdot K^{-4}$

$m_e = 9,10953\times10^{-31}\, kg = 5,48580\times10^{4}\, u.m.a$

$m_p = 1,67265\times10^{-27}\, kg = 1,007276\times10^{4}\, u.m.a$

$R_H = 1,09678\times10^{7}\, m^{-1}$ (experimental)

$R_\infty = \frac{m_e}{4\pi\hbar^3 c}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 = 1,09737\times10^{7}\, m^{-1}$

$E_1(H) = -13,6 eV$

$a_0 = \frac{4\pi\varepsilon_0^2\hbar^2}{m_e e^2} = 5,29177\times10^{-11}\, m$

$c = 3\times10^{8}\, m/s$

$m_e c^2 = 0,511 MeV \quad m_p c^2 = 938 MeV \quad m_n c^2 = 939 MeV$

RADIACIÓ

$R_\nu = I_\nu a_\nu = E_{abs}\,(eq.termic) \quad a + r + t = 1$

Llei de Stefan-Boltzman: $R_T = \sigma T^4 \left[W m^{-2} \equiv J\cdot s^{-1} m^{-2}\right]$

Llei de Planck: $\rho(\nu,T) = \left(\frac{8\pi\nu^2}{c^3}\right)\frac{h\nu}{e^{h\nu/k_B T}-1}$

$\rho(\lambda,T) = \left(\frac{8\pi ch}{\lambda^5}\right)\frac{1}{e^{hc/\lambda k_B T}-1} \quad R_T = \int_0^\infty R_T(\nu)d\nu \quad R_T(\nu) = \frac{c}{4}\rho_T(\nu)$

Llei del desplaçament de Wien:

$\rho_T(\nu) = AT^3 \quad \rho_T(\lambda) = BT^5$

$X_M = \nu_M T^{-1}$

$Z_M = \lambda_M T = 2,898\times10^{-3}\, mK$

EFECTE FOTOELÈCTRIC

Flux energètic: $\varphi_d = \frac{P}{4\pi d^2}$ Potencial de frenada: $V_f = \frac{h}{e}\nu - \frac{W_0}{e}$

Energia de salt: $\frac{1}{2}m_e v_{max}^2 = h\nu - W_0 = eV_0$ Freqüència llindar: $\nu_0 = \frac{W_0}{h}$

EFECTE COMPTON

$\lambda_1 - \lambda_0 = \frac{h}{mc}(1-\cos\theta) \quad \lambda_C = \frac{h}{m_e c} = 0,02426\,\mathring{A}$ Canvi relatiu: $\left|\frac{\Delta\lambda}{\lambda_0}\right|$

$E_{foto} = h\nu \quad P_{foto} = \frac{h\nu}{c}$ **Creació de parells:** $h\nu = m_{e_+} + T_+ + m_{e_-} + T_-$

MODELS ATÒMICS

$\mathrm{K} = \frac{1}{\lambda} = \frac{\nu}{c} = R_\infty Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\forall n_i > n_f$

$n_f = 1$	Lyman
$n_f = 2$	Balmer
$n_f = 3$	Paschen
$n_f = 4$	Bracket
$n_f = 5$	Pfund

Radi de Bohr: $r_n = \frac{4\pi\varepsilon_0\hbar^2 n^2}{Z\mu e^2}$

$E_n = -\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n} = -\frac{\mu}{2\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\frac{1}{n^2} \quad v = \frac{n\hbar}{mr} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{n\hbar}$

$L = \mu v r = n\hbar \quad R_M = R_\infty\frac{\mu}{m} \quad R_{MH} = 10968100 m^{-1}$

$\mu = \frac{mM}{m+M}$ (Normalment: $\mu \simeq m_e$) $\frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1}$

Resolució de l'àtom de Bohr:

$\left.\begin{cases}\frac{\mu v^2}{r} = F(r) = -V'(r) \\ \mu v r = n\hbar\end{cases}\right\} \rightarrow E_n = E_{cin} + E_{pot} = \frac{1}{2}\mu v^2 + V$

$V_{Gravitatori} = -\frac{GmM}{r} \quad V_{Electric} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$

Llei de Moseley: $\frac{1}{\lambda} = R_H(Z-a)^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \rightarrow \frac{1}{\lambda_\alpha} = R_H(Z-1)^2\left(1 - \frac{1}{2^2}\right)$

PROPIETATS ONDULATÒRIES I INDETERMINACIÓ DE HEISENBERG

Hipòtesi de De Broglie: $\lambda = \frac{h}{P} \quad \nu = \frac{E}{h}$

Correcció relativista: $\sqrt{m^2c^4 + c^2|\vec{p}|^2} = mc^2 + T$ amb $T = eV$

Si $T \ll mc^2 \Rightarrow T = \frac{1}{2}mv^2 \quad P = mv \qquad \nu = \frac{c}{\lambda}$

Incerteses: $\Delta x\Delta P \geq \hbar \quad \Delta E\Delta t \geq \hbar \quad \Delta L\Delta\phi \geq \hbar$ Amplada natural: $\Delta\nu = \Delta E/h$

EQ. DE SHRÖDINGER, FUNCIONS DE PROBABILITAT I VALORS ESPERATS

Shrödinguer (DT):

$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t) = E\psi(x,t)$

$-i\hbar\frac{\partial}{\partial t}\psi^*(x,t) = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial x^2}\psi^*(x,t) + V(x)\psi^*(x,t) = E\psi^*(x,t)$

Shrödinger (IDT): $-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V(x)\psi(x) = E\psi(x)$

$\int_{-\infty}^{\infty}\psi(x,t)\psi^*(x,t)dx = \int_{-\infty}^{\infty}|\psi(x,t)|^2 dx = 1 \quad a_n = \int_{-\infty}^{\infty}\phi_n^*(x)\psi(x,0)dx$

$\psi(x,t) = \sum_{n=1}^{\infty}a_n\phi_n(x)e^{iE_n t/\hbar} \quad \sum_n|a_n|^2 = 1 \quad P(E_n) = |a_n|^2 = \int_{-\infty}^{\infty}\phi_n^*(x)\phi_n(x)$

Operadors: $E^{op} = i\hbar\frac{\partial}{\partial t} \quad P^{op} = -i\hbar\frac{\partial}{\partial x} \quad H^{op} = \frac{(P^{op})^2}{2m} + V(x,t)$

Valors esperats: $\langle E^m\rangle = \int_{-\infty}^{\infty}\psi^*(x,t)(E^{op})^m\psi(x,t)dx = \sum_n|a_n|^2(E_n)^m$

$\langle f(E)\rangle = \sum_n|a_n|^2 f(E_n) \qquad \Delta f = \sqrt{\langle f^2\rangle - \langle f\rangle^2}$

$H\psi = E_n\psi \qquad P^{op}\psi(x,t) = P\psi(x,t) \qquad \langle P^2\rangle = 2m\langle E\rangle$

Teorema d'Ehrenfest: $\frac{d\langle x\rangle}{dt} = \frac{\langle P\rangle}{m}$; $\frac{d\langle P\rangle}{dt} = -\left\langle\frac{\partial V}{\partial x}\right\rangle$

BARRERA DE POTENCIAL (amplada a)

$E \leq V_0 \Rightarrow k'^2 = \frac{2m(V_0 - E)}{\hbar^2} = -k'^2 \Leftarrow E \geq V_0 \qquad k = \left[\frac{2mE}{\hbar^2}\right]^{1/2}$

$R = \left[1 + \frac{4E(V_0 - E)}{V_0^2\sinh^2(k_1 a)}\right]^{-1} \qquad T = \left[1 + \frac{V_0^2\sinh^2(k_1 a)}{4E(V_0 - E)}\right]^{-1}$

POU DE PARETS INFINITES (amplada L=2a)

$\phi_n(x) = \begin{cases}\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) & x \in [0,L] \\ 0 & x \notin [0,L]\end{cases} \qquad k = \left[\frac{2m(E - V_0)}{\hbar^2}\right]^{1/2}$

$\phi_n(x) = \begin{cases}\sqrt{\frac{1}{a}}\sin\left(\frac{n\pi(x+a)}{2a}\right) & x \in [-a,a] \\ 0 & x \notin [-a,a]\end{cases} \qquad E_n - V_0 = \frac{\hbar^2\pi^2}{2mL^2}n^2$

POU QUADRAT (amplada L=2a)

$\alpha = \left[\frac{2m(V_0 - E)}{\hbar^2}\right]^{1/2} \qquad k = \left[\frac{2mE}{\hbar^2}\right]^{1/2} \qquad \beta = \left[\frac{2m(V_0 + E)}{\hbar^2}\right]^{1/2}$

$\underbrace{(\alpha a)^2}_{\xi} + \underbrace{(ka)^2}_{\eta} = \theta_0^2 = \left(\frac{2mV_0 a^2}{\hbar^2}\right)$

Parells $\tan\xi = \frac{ka}{\alpha a}$

Senars $ctg\xi = -\frac{ka}{\alpha a}$

$0 \le \theta_0 \le \frac{\pi}{2} \Rightarrow$ 1 solucio

$\frac{\pi}{2} \le \theta_0 \le \pi \Rightarrow$ 2 solucions

$\pi \le \theta_0 \le \frac{3\pi}{2} \Rightarrow$ 3 solucions

$$R = \left[1 + \frac{4E(V_0+E)}{V_0^2 \sin^2(\beta L)}\right]^{-1}$$

$$T = \left[1 + \frac{V_0^2 \sin^2(\beta L)}{4E(V_0+E)}\right]^{-1}$$

OSCIL·LADOR HARMÒNIC

$$\underbrace{\frac{\hbar}{2m}\frac{d^2}{dx^2}}_{\frac{(P^{op})}{2m}}\phi_n + \underbrace{\frac{1}{2}m\omega^2 x^2}_{V(x)}\phi_n = E_n\phi_n \qquad E_n = \left(n+\frac{1}{2}\right)\hbar\omega \quad E_{clas} = \frac{1}{2}m\omega^2 A^2$$

$$\alpha \equiv \left(\frac{2m\omega}{\hbar}\right)^{1/2} \quad \xi \equiv \alpha x \quad \phi_n(x) = \overbrace{\left(\frac{\sqrt{\pi}}{\alpha}2^n n!\right)^{-1/2}}^{A_n} e^{-\frac{1}{2}\xi^2} H_n(\xi)$$

$$\langle x \rangle = \int_{-\infty}^{\infty} \phi_n^*(x) x \phi_n(x) = 0 \quad \int_{-\infty}^{\infty} \phi_n^*(x)\phi_m(x) = \delta_{nm}$$

$$\int_{-\infty}^{\infty} H_n(\xi) H_m(\xi) e^{-\xi^2} d\xi = 2^n n! \sqrt{\pi}\delta_{nm}$$

ÀTOMS AMB UN SOL ELECTRÓ

$$n = 1,2,3,\ldots \quad l = 0,1,2,\ldots,n-1 \quad m = -l,-l+1,\ldots,0,\ldots l$$

Harmònics esfèrics: $Y_{l,m}(\theta,\varphi) = (-1)^n \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}} e^{im\varphi} P_l^m(\cos\theta)$

Polinomis de Legendre: $P_l^m(z) = \frac{(-1)^m}{2^l l!}(1-z^2)^{m/2}\frac{d^{l+m}}{dz^{l+m}}(z^2-1)^l$

$$\Psi(\vec{r}) = R_{n,l}(r) Y_{l,m}(\theta,\varphi) \qquad T(t) = e^{-i\frac{E_n}{\hbar}t} \qquad \int_0^{\infty} |R_{n,l}(r)|^2 dr = 1$$

$$\int_{\Omega} Y_{l',m'}^*(\theta,\varphi) Y_{l,m}(\theta,\varphi) = \delta_{l'l}\delta_{m'm} \qquad P = \int_{R_1}^{R_2} r^2 |R_{n,l}(r)|^2 dr \int_{\Omega} Y^* Y d\Omega$$

$$R_{n,l}(r) = -\sqrt{\left(\frac{2z}{na_0}\right)^3 \frac{(n-l-1)!}{2n}} e^{-\frac{zr}{na_0}} \left(\frac{2r}{na_0}\right)^l L_{n+l}^{2l+1}\left(\frac{2r}{na_0}\right)$$

$$R_{1,0} = 2\left(\frac{z}{a_0}\right)^{3/2} e^{\frac{-z\cdot r}{a_0}} \qquad R_{2,0} = \left(\frac{z}{2a_0}\right)^{3/2}\left(2 - \frac{z\cdot r}{a_0}\right) e^{\frac{-z\cdot r}{2a_0}}$$

$$R_{2,1} = \left(\frac{z}{2a_0}\right)^{3/2} \cdot z\cdot r \frac{1}{a_0\sqrt{3}}\left(2 - \frac{z\cdot r}{a_0}\right) e^{\frac{-z\cdot r}{2a_0}}$$

$$\langle r \rangle = \int_0^{\infty} r P_{n,l}(r) dr = \int_0^{\infty} R_{n,l}^* R_{n,l} dr \quad \langle L_x \rangle = \langle L_y \rangle = 0$$

$$L_{\pm} = L_x \pm i L_y \qquad L_{\pm} Y_{l,m} = \sqrt{l(l+1) - m(m\pm 1)}\hbar Y_{l,m\pm 1}$$

$$L_x = (L_+ + L_-)/2 \qquad L_y = (L_+ - L_-)/(2i)$$

$$\langle L_z \rangle = \sum m_l \hbar |C_{n,l,m}|^2 \quad \langle L^2 \rangle = \sum l(l+1)\hbar^2 |C_{n,l,m}|^2 \quad \langle V(r) \rangle = -\frac{e^2}{4\pi\varepsilon_0}\left\langle \frac{1}{r} \right\rangle$$

ANNEX (integrals més usuals)

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \qquad \int_{-\infty}^{\infty} e^{-ax^2+bx+c} dx = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}+c} \qquad \int_0^{\infty} x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{n+1}}$$

$$\int_0^{\infty} \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15} \qquad \int_{-\infty}^{\infty} x^2 e^{-ax^2} = \frac{\sqrt{\pi}}{2a^{3/2}}$$