Física estadística

$S = k_B \ln \Omega(E,V,N)$

$$\frac{1}{T} = \left(\frac{\partial}{\partial S}E\right)_{V,N} \quad , \quad p = \left(\frac{\partial}{\partial S}V\right)_{E,N}$$

$$\mu = -T\left(\frac{\partial}{\partial S}N\right)_{E,V}$$

$$\Omega = \frac{1}{h^{3N}} \int_{E<H<E+\delta E} \cdots \int \mathrm{d}^{3N}q\mathrm{d}^{3N}p \quad , \ \lambda = \sqrt{\frac{h^2}{2\pi m k_B T}}$$

$$V_n = C_n r^n \ , \quad A_n = nC_n r^{n-1} \ , \quad \to \ C_n = \frac{2}{n}\frac{\pi^{n/2}}{\Gamma(n/2)}$$

$$\text{Gas ideal: } S \approx k_B N \left(\ln\frac{V}{N} + \frac{5}{2}\ln\frac{4m\pi E}{3h^2 N} + \frac{3}{2}\right)$$

$$Z = \sum_{E_t} \Omega_1(E_r)\ \mathrm{e}^{-\beta E_r} = \sum_r \mathrm{e}^{-\beta E_r}$$

$$F(T,V,N) = U - TS = -k_B T \ln Z$$

$$p = -\left.\frac{\partial F}{\partial V}\right)_{T,N} \ , \quad S = -\left.\frac{\partial F}{\partial T}\right)_{V,N} \ , \quad \mu = \left.\frac{\partial F}{\partial N}\right)_{T,V}$$

$$C_v = \left.\frac{\partial \langle E\rangle}{\partial T}\right)_{V,N} = T\left.\frac{\partial S}{\partial T}\right)_{V,N} \ , \quad \langle E\rangle = -\left.\frac{\partial}{\partial \beta}\ln Z\right)_{V,N}$$

$$(\Delta E)^2 = \left(\frac{\partial^2}{\partial\beta^2}\ln Z\right)_{N,V} = -k_B T^2 C_v$$

$$Z(T,V,N) = \frac{1}{N!}Z_1^N = \frac{1}{N! h^{3N}}\int \mathrm{e}^{-\beta H(p,q)}\ \mathrm{d}^{3N}q\ \mathrm{d}^{3N}p$$

$$Z_1(T,V) = \frac{1}{h^3}\int \mathrm{e}^{-\beta H(p,q)}\mathrm{d}^3 q\ \mathrm{d}^3 p$$

$$\left\langle x_i \frac{\partial H}{\partial x_j}\right\rangle = k_B T \delta_{ij}$$

$$H = \sum_{k=1}^{N'} \alpha_k x_k^\eta \to \langle E\rangle = \frac{N' k_B T}{\eta}$$

$$\mathcal{Q}(T,\mu,V) = \sum_{N_s=0}^{\infty}\sum_{E_r}\Omega(E_r,N_s)\mathrm{e}^{-\beta E_r + \beta\mu N_s}$$

$$z = \mathrm{e}^{\mu\nu} = \mathrm{e}^{-\alpha} : \quad \mathcal{Q} = \sum_{N_s=0}^{\infty} z^{N_s} Z(T,V,N_s)$$

$$\mathcal{Q} = \sum_{N_s}\left(zZ_1(T,V)\right)^{N_s} = \frac{1}{1-zZ_1}$$

$$\mathcal{Q} = \sum_{N_s}\frac{1}{N_s!}\left(zZ_1(T,V)\right)^{N_s} = \mathrm{e}^{zZ_1}$$

$$\langle N\rangle = k_B T\left.\frac{\partial}{\partial\mu}\ln\mathcal{Q}\right)_{T,V} = -\left.\frac{\partial}{\partial\alpha}\ln\mathcal{Q}\right)_{T,V} = z\left.\frac{\partial}{\partial z}\ln\mathcal{Q}\right)_{T,V}$$

$$(\Delta N)^2 = k_B T\left.\frac{\partial}{\partial\mu}\langle N\rangle\right)_{T,V} = -\left.\frac{\partial}{\partial\alpha}\langle N\rangle\right)_{T,V} = \frac{\langle N\rangle^2}{\beta\ V}\kappa_T$$

$$\langle U\rangle = -\left(\frac{\partial}{\partial\beta}\ln\mathcal{Q}\right)_{z,V} = k_B T^2\left(\frac{\partial}{\partial T}\ln\mathcal{Q}\right)_{z,V}$$

$$(\Delta U)^2 = k_B T^2 C_v + \left(\frac{\partial}{\partial\langle N\rangle}\langle E\rangle\right)_{T,V}^2 (\Delta N)^2$$

$$\Xi(T,\nu,V) = -k_B T\ln\mathcal{Q} \quad , \quad S = -\left(\frac{\partial\Xi}{\partial T}\right)_{V,\mu}$$

$$\Xi(T,\mu,V) = U - TS - \mu N = -pV$$

Vibr. sol. $Z_1(\omega,T) = \mathrm{e}^{-\beta\hbar\omega/2}/(1-\mathrm{e}^{-\beta\hbar\omega})$

$$\ln Z = \int_{\omega_m}^{\omega_M}\mathrm{d}\omega\ g(\omega)\left(-\beta\omega\hbar/2 - \ln(1-\mathrm{e}^{-\beta\hbar\omega})\right)$$

$$U = \int_{\omega_m}^{\omega_M}\mathrm{d}\omega\ g(\omega)\left(\omega\hbar/2 + \frac{\hbar\omega\mathrm{e}^{-\beta\hbar\omega}}{1-\mathrm{e}^{-\beta\hbar\omega}}\right)$$

$$C_v = k_B\int_{\omega_m}^{\omega_M}\mathrm{d}\omega\ g(\omega)(\beta\hbar\omega)^2\frac{\mathrm{e}^{\beta\hbar\omega}}{(\mathrm{e}^{\beta\hbar\omega}-1)^2}$$

$$\int_{\omega_m}^{\omega_M}\mathrm{d}\omega\ g(\omega) = dN$$

Einstein $g(\omega) = 3N\ \delta(\omega-\omega_E)$

Debye $p = \dfrac{h\omega}{2\pi c}$, $g(\omega)\mathrm{d}\omega = \dfrac{1}{h^d}\mathrm{d}^d q\ \mathrm{d}^d p$, $\omega\in(0,\omega_D)$

Eq. sólido-vapor $\mathcal{Q}_v = \mathrm{e}^{z_v Z_v^1}$, $\mathcal{Q}_s = 1/(1-z_s Z_s^1)$

$$\langle N_v\rangle = \frac{Z_v^1}{Z_s^1} \quad , \quad N_s = N_T - N_v$$

$$N_v(T_c,V) = N_T \ , \quad N_s(T_c,V) = 0 \ , \quad pV = k_B T\frac{Z_v^1}{Z_s^1}$$

Poliatòmicas

$$H = \frac{1}{2}m\dot{R}^2 + \frac{p_r^2}{2\mu} + \frac{L^2}{2\mu r^2} + V(r) \ , \ L^2 = p_\theta^2 + \frac{p_\varphi^2}{\sin\theta}$$

$$\text{Rot. } H = \frac{L^2}{2I} \ , \quad I = \mu r_0^2 \ , \quad \theta_r = \frac{\hbar^2}{2Ik_B} \ , \quad \varepsilon_r = \frac{l(l+1)\hbar^2}{2I}$$

$$g_l = 2l+1 \ , \quad Z_{1,r} = \sum_l (2l+1)\mathrm{e}^{-l(l+1)\frac{\theta_r}{T}}$$

$$\text{cont. } \theta_r \ll T \quad Z_{1,r} = \int_0^\infty \mathrm{d}l\ (2l+1)\mathrm{e}^{-l(l+1)\frac{\theta_r}{T}} = T/\theta_r$$

$$\text{Euler-Mc Laurin}, T^2\ C_v = Nk_B\left(1 + \frac{1}{45}\left(\frac{\theta_r}{T}\right)^2 + \cdots\right)$$

$$\text{Vibr. } H_v = \frac{p_r^2}{2\mu} + \frac{1}{\mu}\omega^2(r-r_0)^2 \ , \quad \theta_v = \frac{\hbar\omega}{k_B} \approx 10^3 K$$

$$Z_{1,v} = \sum e^{-\beta\hbar\omega(n+1/2)} = \frac{e^{-\theta_v/T}}{1-e^{-\theta_v/T}}$$

Gas fotones $\varepsilon = cp = \hbar\omega$, $\omega = kc$, (3D) $g(\omega) = \frac{V\omega^2}{\pi^2 c^3}$, $\nexists\mu(=0)$

$$u(\omega,T) = \frac{\langle n(\omega)\rangle}{V} g(\omega) = \frac{\hbar\omega}{e^{\beta\hbar\omega}-1}\frac{\omega^2}{\pi^2 c^3}$$

$$u(T) = \frac{\pi^2}{15}\frac{k_B^4}{c^3\hbar^3}T^4 = \frac{4}{c}\sigma T^4$$

$$\frac{pV}{k_B T} = \ln\mathcal{Q} = -\int_0^\infty \ln(1-e^{-\beta\hbar\omega})\, g(\omega)\mathrm{d}\omega = \frac{1}{3}u(T)\frac{V}{k_B T}$$

$\left\langle \hat{A}\right\rangle = \mathrm{tr}(\hat\rho\hat A) = \sum \langle\psi_i|\,\hat A\,|\psi_i\rangle$, $\hat\rho = \sum \omega_i\,|\psi_i\rangle\langle\psi_i|$, $\sum\omega_i = 1$

$$\omega_i = \frac{1}{\Omega} \quad , \quad Z_N = \mathrm{tr}\left(e^{-\beta H}\right) = \sum_i e^{-\beta E_i}$$

$$\mathcal{Q} = \mathrm{tr}\left(e^{-\beta(\hat H-\mu\hat N)}\right) = \sum_i e^{-\beta(H_i-\mu N_i)}$$

$$\psi^A_{\mathrm{ferm}} = \frac{1}{\sqrt{N!}}\sum_{P\in S_N}(-1)^{\sigma(P)}\ \varphi_{\varepsilon_1}(\xi_{P(1)})\cdots\varphi_{\varepsilon_N}(\xi_{P(N)})$$

$$\psi^S_{\mathrm{bos}} = \sqrt{\frac{1}{N!\cdot n_1!n_2!\cdots n_N!}}\sum_{P\in S_N}\varphi_{\varepsilon_1}(\xi_{P(1)})\cdots\varphi_{\varepsilon_N}(\xi_{P(N)})$$

$$\mathcal{Q}(T,\mu,V) = \prod_j \mathcal{Q}_j \quad\rightarrow\quad \mathcal{Q}_j = \sum_{n_j}\left(e^{-\beta\varepsilon_j}z\right)^{n_j}$$

$$\mathcal{Q}^{\text{B-E}}_j = \frac{1}{1-ze^{-\beta\varepsilon_j}}\ ,\ \mathcal{Q}^{\text{F-D}}_j = 1+ze^{-\beta\varepsilon_j}\ ,\ \mathcal{Q}^{\text{M-B}} = e^{zZ_1}$$

$$\ln\mathcal{Q} = \frac{1}{a}\sum_j \ln\left(1+az\,e^{-\beta\varepsilon_j}\right)\ ,\quad Q_j = \left(1+az\,e^{-\beta\varepsilon_j}\right)^{\frac{1}{a}}$$

$$\langle n_j\rangle = \frac{1}{z^{-1}e^{\beta\varepsilon_j}+a}\ ,\quad (\Delta n_j)^2 = \frac{\langle n_j\rangle}{1+az\,e^{-\beta\varepsilon_j}}$$

$$q = \ln\mathcal{Q} = \frac{1}{a}\int_{0^+}^\infty \ln\left(1+az\,e^{-\beta\varepsilon}\right)\,g(\varepsilon)\mathrm{d}\varepsilon + \frac{1}{a}\ln(1+az)$$

$$\langle E\rangle = \int_0^\infty \frac{\varepsilon\,g(\varepsilon)\mathrm{d}\varepsilon}{z^{-1}e^{\beta\varepsilon}+a}$$

$$\langle N\rangle = \int_{0^+}^\infty \frac{g(\varepsilon)\mathrm{d}\varepsilon}{z^{-1}e^{\beta\varepsilon}+a} + \frac{z}{1+az}$$

$$(\Delta N)^2 = \int_0^\infty \frac{z^{-1}e^{\beta\varepsilon}\,g(\varepsilon)\mathrm{d}\varepsilon}{\left(z^{-1}e^{\beta\varepsilon}+a\right)^2}$$

Bosones $(a=-1,\ d=2)$, $g(\varepsilon) = A2\pi m/h^2 = g$

$$\mu_{B-E} = -\frac{g}{\beta}\ln(1-e^{-\langle N\rangle\beta/g})\ ,\quad z = 1-e^{-\beta\langle N\rangle/g}$$

$$\langle E\rangle = g\int_0^\infty \frac{\varepsilon\mathrm{d}\varepsilon}{z^{-1}e^{\beta\varepsilon}-1} = \frac{g}{\beta}L_2(1-e^{-\beta\langle N\rangle/g})$$

$$L_2(|z|<1) = \sum_{n=1}\frac{z^n}{n^2}\ ,\quad L_2(-z) = -L_2\left(-\frac{1}{z}\right) - \frac{1}{2}\ln^2 z - \frac{\pi^2}{6}$$

Fermiones $(a=1)$ $\mu_{F-D}(T\to 0) = \varepsilon_F = \langle N\rangle/g$

$$\mu_{F-D} = k_B T\ln(e^{\langle N\rangle\beta/g}-1)\ ,\quad z_{F-D} = -1+e^{\langle N\rangle\beta/g}$$

$$\frac{1}{e^{\beta(\varepsilon-\mu)}+1}\underset{T\to 0}{\longrightarrow}\theta(\varepsilon-\varepsilon_F)$$

$$\langle E(T\to 0)\rangle = g\varepsilon_F^2/2 = \frac{\langle N\rangle^2}{2g}\ ,\quad \langle N(T\to 0)\rangle = g\,\varepsilon_F$$

$$\text{Sommerfeld } T\downarrow\ \langle E\rangle = \frac{g\mu^2(0)}{2} + \frac{\pi^2 g}{6\beta^2} + o(e^{-\beta\mu(T=0)}$$

$$\langle E\rangle = -\frac{g}{\beta^2}L_2(-z)$$

$$\frac{pA}{k_B T} = \frac{1}{a}\int_0^\infty g\ln(1+aze^{-\beta\varepsilon})\mathrm{d}\varepsilon \to pA = \langle E\rangle = \frac{2}{d}\langle E\rangle$$

$E_r = N_A^2/2g$, $T_r = N_A/2k_B g$, $p_r = E_r/A$

$\Pi = p/p_r$, $\omega = (A/\langle N\rangle)/(A/N_A)$, $\tau = T/T_r$

$$\Pi_{M-B} = \tau/\omega\ ,\quad \Pi_a = -\frac{\tau^2}{2a}L_2(1-\varepsilon 2a/\tau\omega)$$

Condensado B-E $s=0$

$$N_0 = \frac{z}{1-z}\ ,\quad g(\varepsilon) = \frac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}$$

$$N_e = \int_0^\infty \frac{\varepsilon g(\varepsilon)}{z^{-1}e^{\beta\varepsilon}-1}\mathrm{d}\varepsilon = \frac{V}{\lambda^3}g_{3/2}(z)$$

$$\langle E\rangle = \frac{3}{2}k_B T\frac{V}{\lambda^3}g_{5/2}(z)\ ,\quad p\beta = g_{5/2}(z)/\lambda^3 \leftarrow pV = \frac{2}{3}\langle E\rangle$$

$$T\to 0\ ,\ z\to 1\ ,\ N_e = \frac{V}{\lambda^3}2.612\cdots$$

$\Pi = p/p_r$, $\omega = V/V_r$, $\tau = T/T_r$, $T_r = T_c(N_A,V_r)$

$$\text{Coex. } \Pi_c\omega_c^{5/3} = 1 \quad ,\quad \lambda^3(T_c) = v_r g_{3/2}(1)\tau_c^{-3/2}$$

$$\text{Vapor } \Pi = \tau^{5/2}\frac{g_{5/2}(z)}{g_{5/2}(1)}\ ,\quad \omega = \tau^{-3/2}\frac{g_{5/2}(z)}{g_{5/2}(1)}$$

$$\text{v-cond } \Pi = \tau^{5/2}\ ,\quad \frac{\mathrm{d}\Pi_c}{\mathrm{d}\tau_c} = \frac{\bar l}{\tau_c\omega_c}\ ,\quad \bar l = \frac{5}{2}\tau_c$$

$$N_0/N = 1-\frac{\lambda^3(T_c)}{\lambda^3(T)} = \left(1-\left(\frac{T}{T_c}\right)^{3/2}\right)\theta(T_c-T)$$

$$g_n(z) = \frac{1}{\Gamma(n)}\int_0^\infty \frac{x^{n-1}}{z^{-1}e^z-1}\mathrm{d}x$$

$$E_m = -\mu_B g_L H m\ ,\quad m = -s,\cdots,s\ ,\quad \mu_B = \frac{e\hbar}{2m_e}$$

$$x = \beta\mu_B g_L H s\ ,\quad Z_1 = \sum_{m=-s}^{s} e^{-xm/s} = \frac{\sinh x(1+1/2s)}{\sinh x/2s}$$

$$M(H,T) = \left(-\frac{\partial F'}{\partial H}\right)_T = M_0 B_s(x)$$

$$= (N g_L \mu_B s)\left[\left(1+\frac{1}{2s}\right)\coth\left(1+\frac{1}{2s}\right)x - \frac{1}{2s}\coth\frac{x}{2s}\right]$$

$$U' = -H M_0 B_s(x) \quad , \quad C_H = \left(\frac{\partial U'}{\partial T}\right)_H = N k_B x^2 B_s'(x)$$

$$H = -\mu H\cos\theta\,, \quad Z_1 = \int \mathrm{d}\varphi \mathrm{d}\theta\, \sin\theta\, \mathrm{e}^{\beta\mu\cos\theta} = 4\pi\frac{\sinh x}{x}$$

$$M = M_0 L(x) = N\mu\left[\coth x - \frac{1}{x}\right]$$

$s \to \infty$, $g_L \to 0$, $g_L s \mu_B \to \mu$, $B_s(x) \to L(x)$

$U(S,V)$		$\mathrm{d}U =$	$T\mathrm{d}S - p\mathrm{d}V + \mu\mathrm{d}N$
$H(S,p)$	$= U + pV$	$\mathrm{d}H =$	$T\mathrm{d}S + V\mathrm{d}p + \mu\mathrm{d}N$
$F(T,V)$	$= U - TS$	$\mathrm{d}F =$	$-\, S\mathrm{d}T - p\mathrm{d}V + \mu\mathrm{d}N$
$G(p,T)$	$= U + pV - TS$	$\mathrm{d}G =$	$V\mathrm{d}p - S\mathrm{d}T + \mu\mathrm{d}N$

Integrales

$$\int_0^\infty x^m e^{-ax^2}\mathrm{d}x = \frac{\Gamma\left(\frac{m+1}{2}\right)}{2a^{\frac{m+1}{2}}} \quad , \quad \int_0^\infty x^n e^{-ax}\mathrm{d}x = \frac{\Gamma(n+1)}{2a^{n+1}}$$

$$\int_0^\infty \frac{x^{n-1}}{e^x - 1}\mathrm{d}x = \Gamma(n)\xi(n) \quad , \quad \Gamma(1/2) = \sqrt{\pi}$$

$$\text{geométrica} \ \sum_{k=0}^{N-1} ar^k = \frac{a(1-r^N)}{1-r} = \frac{a}{1-r}$$

$$\text{Stirling} \ \ln n! \sim n\ln n - n + \ln n + \frac{1}{2}\ln(2\pi)$$

Combinatória

$$VR(N,m) = N^m \quad , \quad VSR(N,m) = \frac{N!}{(N-m)!}$$

$$CR(N,m) = \frac{(N+m-1)!}{m!(N-1)!} \quad , \quad CSR(N,m) = \frac{N!}{m!(N-m)!}$$

Gracias a Sergio Oller por las correcciones.