1.MECÁNICA DE FLUIDOS

HIDROSTÁTICA

Densidad, presión, Ec. Fundamental:

$$\rho = \frac{m}{V}, \quad p = \frac{F}{A}, \quad \frac{\mathrm{d}p}{\mathrm{d}z} = -\rho g \to p = p_o + \overbrace{\rho g h}^{\text{p.hidrost.}}$$

Fuerza en una pared:

$$F_p = \rho g a \cdot \frac{1}{2} H^2$$

Flotación:

$$F_A = mg \Rightarrow \rho_l V_s g = mg$$

Fluido compresible ($H_0 = 8$)

$$p = p_o \cdot e^{\frac{-z}{H_0}}$$

$$\rho = \rho_0 \cdot e^{\frac{-z}{H_0}}$$

Tensión superficial:

$$\sigma \ell = F_r \Rightarrow F = \Delta P \cdot S$$

$$\sigma 2\pi R \cos\theta = mg \Rightarrow h = \frac{2\sigma \cos\theta}{\rho g R}$$

HIDRODINÁMICA

Ec. continuidad:

$$v_1 S_1 = v_2 S_2 = Q$$

Ec. Bernoulli:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 = C$$

Relaciones:

$$v = \sqrt{2gh}; \quad v_1 = \sqrt{\frac{2gh}{\left(\frac{S_1}{S_2}\right)^2 - 1}}; \quad Q = \sqrt{\frac{2gh}{S_1^2 - S_2^2}} S_1 S_2$$

Sifón:

$$v_2^2 = 2g(l + d)$$

Viscosidad:

$$F = \eta S \frac{v}{z}; \quad R = \frac{8\eta L}{\pi r^4}; \quad N_r = \frac{\rho v R}{\eta}$$

Fla. de Poiseuille:

$$v = \frac{\Delta P}{4\eta\ell}(R^2 - r).$$

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2. TERMODINÁMICA

$$T_f = \sum_i \frac{m_i T_i}{m_i}; \quad \frac{c_2}{c_1} = \frac{m_1(t_1 - t_f)}{m_2(t_2 - t_f)};$$

$$\Delta L = \alpha L_0 \Delta T; \quad \Delta V = \beta V_0 \Delta T;$$

$$\Delta Q = m c_e \Delta T = c_c \Delta T; \quad W = \frac{\Delta Q}{\Delta t}$$

TRANSPORTE DE CALOR

1. Conducción:

Flujo:

$$I_T = \frac{\Delta Q}{\Delta t} = \kappa A \frac{T_+ - T_-}{L}; \quad j_T = -\kappa \frac{\mathrm{d}T}{\mathrm{d}x}$$

Conductividad térmica de una pared:

$$j_T = \frac{\kappa}{\ell}\Delta T; \; R_f = \frac{\ell}{\kappa}; \quad I_T = j_T S = \frac{\kappa S}{\ell}(T_0 - T_\ell); \quad \Delta T = I \cdot R_T$$

2 Barras en Serie:

$$\Delta T = I R_{eq}; \quad R_{eq} = \frac{\ell}{S}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right)$$

$$I = \frac{\Delta T}{R_{eq}} = (T_0 - T_2) \cdot \frac{S}{\ell}\left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)$$

2 Barras en Paralelo:

$$\Delta T_1 = \Delta T_2 = \Delta T; \quad I = I_1 + I_2; \quad \Delta T = I_i R_i$$

$$\Delta T = R_{eq} I$$

Aislamiento:

$$\ell = \sqrt{\alpha \Delta T}; \quad \alpha = \frac{\kappa}{\rho_c}$$

$$c = \frac{\kappa_2}{\kappa_1}\frac{\ell_1}{\ell_2} = \sqrt{\frac{\rho_2 c_2 \kappa_2}{\rho_1 c_2 \kappa_1}}$$

$$R = \frac{d}{\kappa S}$$

2. Convección:

Ley de enfriamiento de Newton:

$$\frac{\Delta Q}{\Delta t} = qS[T_S - T_\infty];$$

$$R_{conv} = \frac{\ell}{qS}; \quad R_{eq} = \sum_i R_i$$

Flujo y Fórmula de Wien:

$$j = \kappa \frac{\Delta T}{\ell}; \quad \lambda_m T = A$$

3. Radiación:

$$I_R = \frac{I_E}{I_A}$$

Fórmula de S. Boltzmann:

$$j_T = \frac{2\pi^5}{15}\frac{K_B^4 T^4}{h^3 c^2}$$

Pérdida de calor:

$$\left.\frac{\Delta Q}{\Delta t}\right|_{\text{neta}} = e\sigma S(T_s^4 - T_\infty^4)$$

Gases Ideales

$$pV = nRT; \quad k_B = \frac{R}{N_A}; \quad n = \frac{m}{\mu}; \quad \mu = m_0 N_A$$

$$pV = N k_B T = n N_a k_B T = nRT$$

Procesos: Isotérmicos, Isobáricos e Isocóricos:

$$pV = cte. \quad V = V_0(1 + \alpha T); \quad p = p_0 \alpha T$$

Cálculo cinético de la presión:

$$p = \frac{1}{3}\rho \bar{v}^2$$

Energía cinética media:

$$\bar{E} = \frac{3}{2}k_B T; \quad \bar{v}^2 = \frac{3k_B T}{m} \to v_{rcm} = \sqrt{\frac{3k_B T}{m}}$$

Velocidad media at.:

$$\bar{v}_{at} = \frac{(R_2 - R_2)\omega R_2}{S}$$

Distribución de Maxwell (Boltzman):

$$f(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{\frac{-mv2}{2k_B T}}$$

Velocidad máxima, velocidad media y v_{rcm}:

$$v_M = \sqrt{\frac{2k_B T}{m}}; \quad \bar{v} = \sqrt{\frac{8k_B T}{\pi M}}; \quad v_{rcm} = \sqrt{\frac{3k_B T}{m}}$$

Relación $f(v) \to f(\varepsilon)$

$$f(\varepsilon) = \frac{2}{\sqrt{\pi}} \frac{1}{(k_B T)^{3/2}} \sqrt{\varepsilon} e^{\frac{-\varepsilon}{k_B T}}$$

Conclusión: $Q = c\mu n \Delta T$
I Ley de la termodinámica:

$$Q = \Delta U + W \Rightarrow dQ = \mathrm{d}U + p\mathrm{d}V$$

$$C_\mu \mathrm{d}T = dU_\mu + R\mathrm{d}T \Rightarrow C_p = C_v + R$$

Th. de equipartición de energía i recorrido libre:

$$c_1 = i\frac{1}{2}k_B \Longrightarrow c_\mu = i\frac{1}{2}R; \quad \lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

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3. ONDAS Ecuación de un oscilador armónico (MAS):

$$\ddot{x} + \omega^2 x = 0; \quad \omega = 2\pi\nu; \quad T = 2\pi\sqrt{\frac{m}{k}}; \quad x = A\cos(\omega t + \varphi_0);$$

Energía:

$$E = \frac{1}{2}mv2 + \frac{1}{2}kx^2 \Longrightarrow m\ddot{x} + kx = 0$$

$$E = \frac{1}{2}kA^2$$

Péndulo simple:

$$m\ddot{x} = mg\sin\alpha; \quad x = L\alpha; \quad T = 2\pi\sqrt{\frac{L}{g}}$$

Tubo de U:

$$\Delta F = pS - \rho g S \Delta H$$

Suma de 2 M.A.S:

$$x = x_1 + x_2 = 2A\cos\frac{\varphi}{2}\cos\left(\omega t + \frac{\varphi}{2}\right)$$

Oscilación periódica: $y(t_0 + T) = y(T)$
Oscilación armónica: $y(t) = A\cos(\omega t + \varphi_0)$
Función de onda: $y = f(x - vt)$
Relaciones:

$$\lambda = vt \to v = \lambda\nu \to \omega = kv; \quad v = \sqrt{\frac{F_T}{\mu}}; \quad k = \frac{2\pi}{\lambda}$$

$$a_{max} = A\omega^2; \quad v_{max} = A\omega$$

Función de onda armónica:

$$y(x,t) = A\sin(kx \pm \omega t + \varphi_0)$$

$$v_y(x,t) = A\omega\sin(kx - \omega t)$$

$$a_y(x,t) = -A\omega\cos(kx - \omega t) = -\omega^2 y(x,t)$$

Energía, potencia e intensidad de una onda:

$$E_c = \frac{1}{2}\mu^2 v_y^2 = \frac{1}{4}\mu A^2$$

$$E = E_c + U_p = \frac{1}{2}\mu A^2\omega^2 = \frac{1}{2}\mu v_{\max}^2$$

$$\Delta E = \frac{1}{2}\mu A^2\omega^2 v\Delta t$$

$$P = \frac{\Delta E}{\Delta t} = \frac{1}{2}\mu A^2\omega^2 v$$

$$P = \frac{1}{2}\mu v\omega^2 A^2 \to \text{cuerda}$$

$$P = \frac{1}{2}\rho v\omega^2 A^2 \to \text{medio volum.}$$

$$I = \frac{\Delta E}{\Delta S\Delta t} = \frac{P}{\Delta S} = \frac{1}{2}\rho v\omega^2 A^2$$

Ondas estacionarias, principio de superposición:

$$y_T = y_1 + y_2 = 2A\cos(kx)\cos(\omega t)$$

$$A_i + A_r = A_t; \quad A_i k_1 - A_r k_2 = A_t k_2$$

$$\alpha = \frac{v_2}{v_1} = \frac{k_1}{k_2}; \quad \omega = k_1 v_1 = k_2 v_2; \quad \alpha = \frac{\rho v_2}{\rho v_1}$$

Desfases: (1)temoral, (2) $= y \neq t$, (3) $\neq y = t$:

$$\Delta t = \frac{\Delta x}{v}; \quad \Delta\varphi = \omega\Delta t; \quad \Delta\varphi = k\Delta x$$

Coeficientes de transmisión, reflexión; potencia, coeficiente de reflectividad y transmitancia:

$$t = \frac{A_t}{A_i} = \frac{2\alpha}{\alpha + 1}; \quad r = \frac{A_r}{A_i} = \frac{\alpha - 1}{\alpha + 1}$$

$$P = \frac{1}{2}\mu v\omega^2 A^2 = \frac{1}{2}F_T\omega^2\frac{A^2}{v}; \quad R = \frac{P_r}{P_i} = \frac{A^2 r}{A^2 i} = \frac{(\alpha - 1)^2}{(\alpha + 1)^2} = r^2$$

$$T = \frac{P_t}{P_i} = \frac{A_t^2/v_2}{A_i^2/v_1} = \frac{t^2}{\alpha} = \frac{4\alpha}{(1+\alpha)^2}; \quad R + T = 1$$

Armónicos(a) extr.fijos; (b) 1 extr. libre:

$$(a) L = n\frac{\lambda_n}{2}; \quad \lambda_n = \frac{2L}{n}; \quad \nu_n = n\frac{v}{2L} = n\nu_1$$

$$y_n(x,y) = 2A\sin(k_n x)\sin(\omega_n t)$$

$$(b) L = n\frac{\lambda_n}{4}; \quad \lambda_n = \frac{4L}{n}; \quad \nu_n = n\frac{v}{4L} = n\nu_1; n = 1, 3, 5...$$

Ondas sonoras:

$$S(x,t) = S_n\cos(kx - \omega t); \quad \Delta p(x,t) = \Delta p_n\sin(kx - \omega t)$$

$$\Delta P_m = \rho v\omega S_m; \quad v = \sqrt{\frac{B}{\rho}} = \sqrt{\gamma\frac{RT}{\mu}} = \sqrt{\gamma\frac{k_B T}{m}}$$

$$I = \frac{1}{2}\frac{(\Delta p_m)^2}{(\rho v)^2}; \quad \beta = 10\log\frac{I}{I_0}$$

Efecto Doppler (1) OMFR, (2) ORFM, (3)OMFM:

$$(1)\nu' = \nu\left(1 \pm \frac{v_{ob}}{v}\right); \quad (2)\nu' = \nu\left(\frac{1}{1 \pm \frac{v_f}{v}}\right)$$

$$(3{,}1): \nu' = \nu\frac{v \pm v_{ob}}{v \mp v_f}; \quad (3{,}2)\nu' = \nu\frac{v - v_{ob}}{v + v_f}$$